Examination I
October 1, 2008

1. Given $f(x,y)$

$$f(x,y)=\frac{36}{64}-\frac{9}{64}x^2-\frac{9}{64}y^2+\frac{9}{64}x^2y^2$$

Defined over $x,y\in[-2,2]$.

   a. Is this a valid distribution function? (5 Points)
   b. Derive the marginal distribution functions for x and y. (10 Points)
   c. Are these variables independent? (5 Points)
   d. Compute the conditional distribution function for x given $y=0$. (5 Points).

2. Given $f(x)=\frac{3x^2}{125}$ for $x\in[0,5]$ and $y=4-3x$:

   a. What are the conditions for deriving $g(y)$ based on $y=\phi(x)$? (5 Points).
   b. Derive the distribution function $g(y)$ including the bounds of the new random variable. (10 Points)

3. Using the distribution $f(x,\lambda)=\lambda e^{-\lambda x}$ for $x\in[0,\infty)$:

   a. Derive the moment generating function for x. (10 Points)
   b. Compute the first three moments for this distribution. (10 Points)
   c. Using the random variables in Table 1, calculate the first three sample moments. Do you think that these random variables are from $f(x,3/10)$? (10 Points)

4. Given

$$x\sim N\left(\begin{pmatrix}3\\-2\end{pmatrix},\begin{bmatrix}6&-2\\-2&3\end{bmatrix}\right)$$

   a. Compute the conditional expectation of x_2 such that $x_1=2$. (10 Points)
   b. Compute the conditional variance for x_2 for this value. (10 Points)
   c. Compute the probability that $x_2\in[-1,1]$ given that $x_1=2$. (10 Points)

Table 1. Sample Data

Obs	x	x^2	x^3
1	2.6023	6.7722	17.6237
2	12.1846	148.4655	1808.9986
3	7.0841	50.1851	355.5189
4	2.7195	7.3959	20.1133
5	3.0698	9.4236	28.9285
6	0.0243	0.0006	0.0000
7	3.9935	15.9484	63.6906
8	0.4810	0.2313	0.1113
9	1.2821	1.6437	2.1073
10	10.2083	104.2098	1063.8073
11	8.1042	65.6776	532.2621
12	7.2829	53.0400	386.2831
13	1.5129	2.2887	3.4625
14	4.6757	21.8617	102.2179
15	2.1068	4.4386	9.3514
16	3.2551	10.5954	34.4885
17	0.2683	0.0720	0.0193
18	0.4162	0.1732	0.0721
19	3.5187	12.3812	43.5657
20	1.2607	1.5893	2.0036
21	0.0512	0.0026	0.0001
22	2.3821	5.6746	13.5176
23	0.7567	0.5725	0.4332
24	1.1413	1.3027	1.4868
25	2.1373	4.5680	9.7630
26	2.5630	6.5692	16.8371
27	1.2977	1.6841	2.1855
28	4.3917	19.2869	84.7021
29	6.8388	46.7696	319.8497
30	2.5112	6.3061	15.8360
	100.1221	609.1303	4939.2369

Table 2. Normal Distribution Function

z	$F(z)$	$1-F(z)$	$f(z)$
0.00	0.5000	0.5000	0.3989
0.05	0.5199	0.4801	0.3984
0.10	0.5398	0.4602	0.3970
0.15	0.5596	0.4404	0.3945
0.20	0.5793	0.4207	0.3910
0.25	0.5987	0.4013	0.3867
0.30	0.6179	0.3821	0.3814
0.35	0.6368	0.3632	0.3752
0.40	0.6554	0.3446	0.3683
0.45	0.6736	0.3264	0.3605
0.50	0.6915	0.3085	0.3521
0.55	0.7088	0.2912	0.3429
0.60	0.7257	0.2743	0.3332
0.65	0.7422	0.2578	0.3230
0.70	0.7580	0.2420	0.3123
0.75	0.7734	0.2266	0.3011
0.80	0.7881	0.2119	0.2897
0.85	0.8023	0.1977	0.2780
0.90	0.8159	0.1841	0.2661
1.00	0.8413	0.1587	0.2420
1.05	0.8531	0.1469	0.2299
1.10	0.8643	0.1357	0.2179
1.15	0.8749	0.1251	0.2059
1.20	0.8849	0.1151	0.1942
1.25	0.8944	0.1056	0.1826
1.30	0.9032	0.0968	0.1714
1.35	0.9115	0.0885	0.1604
1.40	0.9192	0.0808	0.1497
1.45	0.9265	0.0735	0.1394
1.50	0.9332	0.0668	0.1295
1.55	0.9394	0.6060	0.1200
1.60	0.9452	0.0548	0.1109
1.65	0.9505	0.0548	0.1023
1.70	0.9554	0.0446	0.0940
1.75	0.9599	0.0401	0.0863
1.80	0.9641	0.0359	0.0790
1.85	0.9678	0.0322	0.0721
1.90	0.9713	0.0287	0.0656
2.00	0.9772	0.0228	0.5400
2.05	0.9798	0.0202	0.0488
2.10	0.9821	0.0179	0.0440
2.15	0.9842	0.0158	0.0396
2.20	0.9861	0.0139	0.0355
2.25	0.9878	0.0122	0.0317
2.50	0.9938	0.0062	0.0175

Table 3. Cumulative Probabilities for the Chi-Square Distribution

Degrees of Freedom	Cumulative Probability						
	0.01	0.05	0.1	0.5	0.9	0.95	0.99
1	0.000157	0.00393	0.0158	0.455	2.71	3.8400	6.6300
2	0.0201	0.103	0.211	1.39	4.61	5.9900	9.2100
3	0.115	0.352	0.584	2.37	6.25	7.8100	11.1000
4	0.297	0.711	1.06	3.36	7.78	9.4900	13.3000